MATH 211.3 Winter Term 2024 Assignment

Assignment #11

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**Problem 1**

**A close-up of a paper with mathematical equations

Description automatically generated**

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Description automatically generated**

**1**

clear;

clc;

matrices = {

[10, -12, -6; 5, -5, -4; -1, 0, 3],

[-14, 20, 10; -19, 27, 12; 23, -32, -13],

[8, -8, -4; 12, -15, -7; -18, 26, 12],

[12, -4, -2; 19, -19, -10; -35, 52, 27]

};

num\_simulations = 100;

for i = 1:length(matrices)

[eigenvector, rayleigh\_quotient] = power\_iteration(matrices{i}, num\_simulations);

fprintf('Matrix (%c):\n', 'a' + i - 1);

fprintf('Dominant Eigenvector:\n');

disp(eigenvector);

fprintf('Rayleigh Quotient (Dominant Eigenvalue):\n');

disp(rayleigh\_quotient);

fprintf('\n');

end

function [eigenvector, rayleigh\_quotient] = power\_iteration(A, num\_simulations)

b\_k = rand(size(A, 1), 1);

for i = 1:num\_simulations

b\_k1 = A \* b\_k;

b\_k = b\_k1 / norm(b\_k1);

end

eigenvector = b\_k;

rayleigh\_quotient = (b\_k' \* A \* b\_k) / (b\_k' \* b\_k);

end

**>>**

**Matrix (a):**

**Dominant Eigenvector:**

**-0.5774**

**-0.5774**

**0.5774**

**Rayleigh Quotient (Dominant Eigenvalue):**

**4.0000**

**Matrix (b):**

**Dominant Eigenvector:**

**0.5774**

**0.5774**

**-0.5774**

**Rayleigh Quotient (Dominant Eigenvalue):**

**-4.0000**

**Matrix (c):**

**Dominant Eigenvector:**

**-0.5774**

**-0.5774**

**0.5774**

**Rayleigh Quotient (Dominant Eigenvalue):**

**4.0000**

**Matrix (d):**

**Dominant Eigenvector:**

**0.5774**

**0.5774**

**-0.5773**

**Rayleigh Quotient (Dominant Eigenvalue):**

**10.0000**

**3**

clear;

clc;

matrices = {

[10, -12, -6; 5, -5, -4; -1, 0, 3],

[-14, 20, 10; -19, 27, 12; 23, -32, -13],

[8, -8, -4; 12, -15, -7; -18, 26, 12],

[12, -4, -2; 19, -19, -10; -35, 52, 27]

};

shifts = [5, 4, 1, 8];

num\_simulations = 100;

for i = 1:length(matrices)

[eigenvector, eigenvalue] = inverse\_power\_iteration(matrices{i}, num\_simulations, shifts(i));

fprintf('Matrix (%c):\n', 'a' + i - 1);

fprintf('Estimated Eigenvector:\n');

disp(eigenvector);

fprintf('Estimated Eigenvalue:\n');

disp(eigenvalue);

fprintf('\n');

end

function [eigenvector, eigenvalue] = inverse\_power\_iteration(A, num\_simulations, mu)

n = size(A, 1);

B = A - mu \* eye(n);

b\_k = rand(n, 1);

for i = 1:num\_simulations

b\_k1 = B \ b\_k;

b\_k = b\_k1 / norm(b\_k1);

end

eigenvalue = (b\_k' \* A \* b\_k) / (b\_k' \* b\_k) + mu;

eigenvector = b\_k;

end

**Matrix (a):**

**Estimated Eigenvector:**

**-0.5774**

**-0.5774**

**0.5774**

**Estimated Eigenvalue:**

**9**

**Matrix (b):**

**Estimated Eigenvector:**

**-0.0000**

**0.4472**

**-0.8944**

**Estimated Eigenvalue:**

**7.0000**

**Matrix (c):**

**Estimated Eigenvector:**

**0.8165**

**0.4082**

**0.4082**

**Estimated Eigenvalue:**

**3.0000**

**Matrix (d):**

**Estimated Eigenvector:**

**0.8165**

**0.4082**

**0.4082**

**Estimated Eigenvalue:**

**17**

**Problem 2**

**A close-up of a paper

Description automatically generated**

**3**

clear;

clc;

matrices = {

[-1, 1, 3; 3, 3, -2; -5, 2, 7],

[-3, -1, 1; 5, 3, -1; -2, -2, 0]

};

eigenvalues\_a = shifted\_qr\_algorithm(matrices{1});

eigenvalues\_d = shifted\_qr\_algorithm(matrices{2});

disp('Eigenvalues for matrix (a):');

disp(eigenvalues\_a);

disp('Eigenvalues for matrix (d):');

disp(eigenvalues\_d);

function eigenvalues = shifted\_qr\_algorithm(A, max\_iterations)

if nargin < 2

max\_iterations = 100;

end

n = size(A, 1);

Ak = A;

eigenvalues = zeros(n, 1);

for i = 1:max\_iterations

mu = Ak(n, n);

[Q, R] = qr(Ak - mu \* eye(n));

Ak = R \* Q + mu \* eye(n);

if i > 1 && all(all(abs(Ak - triu(Ak)) < 1e-8))

break;

end

end

eigenvalues = diag(Ak);

end

**Eigenvalues for matrix (a):**

**2.9997**

**3.0001**

**3.0002**

**Eigenvalues for matrix (d):**

**2.0000**

**-2.0000**

**0.0000**

**5**

clear;

clc;

matrix\_a = [4, 3, 1; -5, -3, 0; 3, 2, 1];

matrix\_d = [11, 4, -2; -10, 0, 5; 4, 1, 2];

eigenvalues\_a = qr\_algorithm(matrix\_a);

eigenvalues\_d = qr\_algorithm(matrix\_d);

disp('Eigenvalues for matrix (a):');

disp(eigenvalues\_a);

disp('Eigenvalues for matrix (d):');

disp(eigenvalues\_d);

function eigenvalues = qr\_algorithm(A, max\_iterations)

if nargin < 2

max\_iterations = 1000;

end

Ak = A;

n = size(A, 1);

for i = 1:max\_iterations

[Q, R] = qr(Ak);

Ak = R \* Q;

end

eigenvalues = diag(Ak);

end

Eigenvalues for matrix (a):

2.0000

0.5000

-0.5000

Eigenvalues for matrix (d):

2.7407

6.2290

4.0303